

Selecting a Representative using Resources, Supports and Oppositions in Agents's Competition

Guillermo De Ita¹, Mireya Tovar², Pedro Bello³ and Meliza Contreras⁴

Computer Sciences Dpto., Universidad Autónoma de Puebla

Contact: ¹deita,²mtovar,³pbello,⁴mcontreras@cs.buap.mx

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Abstract. We address the problem of modeling the selection of a representative among a set of intelligent agents. We design a novel model based on that the candidacy of an agent depends on a set of resources as well as the opinion of each agent about the other agents, in such a way that the candidacy will be for the agent who has a winning strategy (subset of adequate resources) and he obtains a minimum penalty to apply his strategy. Then, the more suitable agent to be a representative is the one who does not generate a lot of opposition into the multi-agent system and gets a minimum penalty for using his winning strategy. This case is an example of a congestion game so that, we can obtain a pure Nash equilibrium which ensures that we can find the representative of the group.

1 Introduction

Artificial Intelligence (AI) has motivated to researchers to explore new reasoning issues and methods, and to combine disparate reasoning modalities into a uniform unified framework, so as to deal with incomplete, imprecise, contradictory, and changing information. Classical logic has been developed long time ago to study unchanging mathematical objects, being well-founded and consistent. It thereby acquired a static character.

However, the paradigm agent demands to represent, besides of static knowledge, dynamic knowledge, too. In this sense, logic theories have claimed a major role in defining the trends of modern research, increasing its influence in the development of algorithms that consider dynamic and interactive knowledge process.

As more and more commercial transactions are performed on networks, there is more interest in designing smart agents that can perform specific actions. The use of intelligent agents has provoked a great commercial interest, and they have been useful for making decisions. Beyond a static knowledge, the agent paradigm attracts, represents and handles dynamic knowledge.

For example, one important human task has been the selection of a representative among a group of people, and different methods have been designed to carry out this task.

In [14] introduce a new class of games. Congestion Games with Failures (CGFs). In a basic CGF (BCGF) agents share a common set of facilities (service providers), where each service provider (SP) may fail with some known probability. For reliability reasons, an agent may choose a subset of the SPs in order to try and

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perform his task. The cost of an agent for using any SP is a function of the total number of agents using this SP. A main feature of this setting is that the cost of an agent for successful completion of his task is the minimum of the costs of his successful attempts. However this multiagent system always possess a pure-strategy Nash equilibrium.

The selection of a representative is an important and common problem wherever it is necessary to choose candidates from political parties, for selecting a department's boss, for selecting a supervisor, etc. In general, when it is necessary to select the best candidate (in accordance with a set of rules) from a group of people.

Work on intelligent agents has reformulated the methods in the area of automatic reasoning [1]. In our case, we are interested in developing deliberative agents that have domain knowledge in the hope of achieving a specific goal.

One of the most common criterion for choosing representatives is the results obtained for direct popular voting. In this case, the candidate who obtains the maximum number of supportive votes is chosen as the representative from the group. This method is adequate when the selection of the representative is based on the popularity of the candidates.

We present here a novel logical model which try to capture some important aspects of the process of selecting a representative from among a group formed by intelligent agents which compete for the candidacy of the group. We assume that the candidacy of an agent depends on a set of 'influences-resources', in such a way that the candidacy is for the agent who obtains a winning strategy (subset of resources). Furthermore, the representative must obtain a minimum penalty for using his winning strategy.

Associated with each resource r there is a congestion function f_r which is a penalty function for sharing such resources among the agents. Besides each agent establishes a set of supportive and opposing constraints in accordance with his beliefs about the candidacy of the other agents. The union of the beliefs of the agents forms a Knowledge Base for the multi-agent system.

Searching for an optimal interactive strategy is a hard problem because its effectiveness depends mostly on the strategies of the other agents involved [2]. However, the agents are autonomous, hence their strategies are private. Thus, we are modeling the selection of a representative as a non-cooperative game where there is a well-defined set of winning strategies and a set of weighted resources for forming the strategies [10].

This model can be seen as an example of a congestion game [6, 9, 12], so that we can obtain a pure Nash equilibrium which ensures the rational outcomes of the game, and at this point, no agent could benefit by unilaterally deviating for his strategy, given the other agents follow their own strategies. Then, arriving at the Nash pure equilibrium we can determine the agent with a winning strategy and with a minimum penalty for using such winning strategy. That agent will be the representative of the multi-agent system.

2 Building the logical model for selecting a representative

We design the selection model considering the Knowledge Base of the beliefs of a multi-agent system and the competitive resources. These elements are the base to form the strategies of the agents in order to obtain the representative position of the multi-agent system.

2.1 Building the Knowledge Base of the beliefs of the multi-agent system

A widely accepted framework for reasoning in intelligent systems is the knowledge-based system approach. The main idea is to represent knowledge, for example the knowledge or beliefs of a multi-agent system, in an appropriate structure or language with a well defined meaning assigned to these beliefs. We show in this section, how to build the KB associated with our multi-agent system.

Let $A = \{A_1, A_2, \dots, A_n\}$ be a set of n intelligent agents. We denote with A_i any agent of A . Let $v(\Sigma)$ the set of variables (agents) of the KB Σ , defined as: $v(\Sigma) = \{A_i \in A: A_i \text{ appears in } \Sigma\}$, while we will denote by Lit the set of literals of a given set. For example, $\text{Lit}(\Sigma) = \{A_i, \neg A_i: A_i \in A\}$. A belief of an agent is a binary implication, so in each implication one literal appears as antecedent and one literal appears as succedent. There are two classes of beliefs: oppositions and supports.

The syntax for an opposing constraint is: $A_i \rightarrow \neg (A_j) (W_{ij})$, indicating that the agent A_i believes that the agent A_j is not a good candidate, and A_i gives a weight of W_{ij} as a punishment. In general, an opposing constraint always has a negative literal as a succedent.

On other hand, the syntax for a supportive constraint, is: $\neg(A_i) \rightarrow A_j (W_{ij})$, indicating that the agent A_i believes that if he is not selected as candidate then the agent A_j could be a good candidate, and A_i gives a weight of W_{ij} as support for the candidacy of A_j . The other kind of supportive constraint has a syntax: $A_i \rightarrow A_j (W_{ij})$ which means that the agent A_i believes that an appropriate companion to his candidacy is the agent A_j , and A_i gives a support of W_{ij} . In general, a supportive constraint is an implication where in the succedent always appears one positive literal.

Each agent contributes with their beliefs, expressed as a set of opposing weighted constraints (oppositions) and a set of supportive weighted constraints (supports) and the union of all beliefs of the agents of A builds the Knowledge Base Σ associated with the multi-agent system.

For any agent $A_i \in A$, the sum of its supportive weights cannot exceed a bound B , and the same bound cannot be exceeded by the sum of its opposing weights. The bound B is common to all agents such that the beliefs of one agent are as important as the beliefs of any other agent.

Let $G_\Sigma = (V, E)$ be the dependence graph generated by the constraints of Σ , being $V = \text{Lit}(A)$ and E the set of constraints given by the agents. Let " \Rightarrow " be the reflexive and transitive closure of an implication: \rightarrow , beginning over any literal of Σ . $T(x)$ denotes the set of all such literals y forced by x , $\forall x \in \text{Lit}(\Sigma)$, $T(x) = \{y \in \text{Lit}(\Sigma) : x \Rightarrow y\}$. For any literal $x \in \text{Lit}(\Sigma)$, we call $T(x)$ the closure of x .

Let $W(e)$ be the weight associated with any edge e in G_{Σ} and let $\text{in_edge}(Y) = \{X \rightarrow Y\}$ be the set of input edges to the node Y of G_{Σ} . We define the support for the candidacy of an agent A_i as: $\text{Support}(A_i) = \sum_{e \in \text{in_edge}(A_i)} W(e)$, and the opposition to the candidacy of an agent A_i as: $\text{Opposition}(A_i) = \sum_{e \in \text{in_edge}(\neg A_i)} W(e)$. Then, the Popularity of the candidacy of an agent $A_i \in A$, is computed, as: $\text{suit}(A_i) = \text{Support}(A_i) - \text{Opposition}(A_i)$.

2.2 Using Competitive Resources

In the selection of a representative among a group of intelligent agents is common that beyond of the beliefs of the agent, the selection of a representative also depends on the set of resources and influences that one agent has and that he uses in order to take advantages over the other agents. We extend our initial logical model for consider a set of competing resources among the agents. This approach has been extensively used in game theory, mainly for congestion games.

Some examples of the type of resources that an agent hopes to get, could be: r_1 : A favorable opinion and support of the actual president of the company, r_2 : A favorable opinion and support of the actual boss of the department, r_3 : Economic support for the competition, r_4 : To take an adequate background for the position, r_5 : To take adequate experience in the prospective position, etc.

We represent the list of resources via a discrete set $R = \{r_1, \dots, r_m\}$. Each resource has assigned a relative value with respect to the other resources. Let W_i be the relative value (weight) associated with the resource $r_i \in R$, $i=1, \dots, m$, and let $W_k =$

$$\sum_{i=1}^m W_i \text{ be the total sum of these weights.}$$

Usually, each agent $A_i \in A$ believes that he has (or he can get) a subset of possible resources $s \subseteq R$. Such a subset s is called a strategy of the agent A_i . Each agent A_i determines a finite set S_i of possible strategies which he can use in order to win the candidacy of the group. Indeed, only some combinations of resources permit reaching the representative position. Let $MC = \{C_1, \dots, C_k\}$ be the set of strategies which permit win the candidacy. We call to any $C_i \in MC$, $i=1, \dots, k$ a *winning strategy*. Then, the agent who has any (or a superset of) $C_i \in MC$ in his strategies set might reach the representative position.

Of course, no agent knows ahead of time which is the winning strategies. In this model is assumed that there is someone, distinct from the agents, who establishes the winning strategies and assigns the relative weights W_i , $i=1, \dots, m$ to each resource. In practice, this responsibility lies on the general director of the company or the human resources department's boss, etc.

Graphically, we can represent this model through a directed graph G_R . G_R has a distinguished node CK (a sink of the graph) representing the candidacy of the group and 2^m nodes more, as well as directed edges linking these nodes. A path in G_R represents one element of the power set $\wp(R)$ then: there are 2^m different paths in G_R . Over these paths, there are minimal paths $MC \in \wp(R)$, called the 'winning

strategies', which permit of reaching the distinguished node CK. Of course, any superset of MC also gives us a path for reaching CK.

We consider that each resource $r_j \in R$, $j=1, \dots, m$ has a personal nature, that is, it is difficult for the same resource to be shared among two or more agents. Then, there is a nondecreasing *congestion function* f_j associated with each resource r_j , $f_j: \{1, \dots, n\} \rightarrow Z$. For this reason, the directed graph G_R is called a *congestion network* [6,9,15]. We illustrate a case of congestion network in figure 1.

For example, a way to characterize f_j could be: $f_j(n_A) = n_A * W_j$, which means, that while the number of agents (n_A) use the same resource r_j , there will be a cost of $n_A * W_j$ for sharing the resource r_j among them, in such a way that as more agents use the same resource, the payoff for using it will be greater.

Each agent A_i chooses one of his strategies $s_i \in S_i$, forming in this way a state (an action in the multi-agent system) $e = (s_1, \dots, s_n) \in S_1 \times \dots \times S_n$, so that, there is a states set $E = \{e_1, \dots, e_k\}$ where each e_j , $j=1, \dots, m$ is an action in the multi-agent system.

For each action $e = (s_1, \dots, s_n) \in E$ we determine n_r , $r=1, \dots, m$ which represents the number of agents whose have chosen the resource r in their strategy. Then, the penalty function (payoff) associated with the agent A_i with chosen strategy s_i is denoted by $Opposition(A_i)$ and it is defined as:

$$Opposition(A_i) = \sum_{r \in s_i} f_r(n_r) + reach(s_i) - suit(A_i)$$

When the function $reach(s_i)$ returns the value: $(-W_k)$ if the path s_i reaches the sink node C_k ; otherwise it is zero. So, in case that $reach(s_i)$ allows to A_i to arrive at the node CK then the value $-W_k$ decreases his penalty because A_i has obtained a feasible solution of the congestion network and where $suit(A_i)$ is the popularity of the candidacy of an agent A_i .

Given any state $e = (s_1, \dots, s_n) \in E$, an *improvement step* of an agent A_i is a change of his strategy from s_i to s'_i such that his $Opposition(A_i)$ decrease. Thus, we can see the neighborhood of a state e consists of those states that deviate from e only in one agent's strategy.

Intuitively, each agent A_i can choose one strategy from among the set of strategies S_i available to him, but his penalty depends on the strategy choices for all agents. Since, in order to compute the cost incurred by A_i we add the cost of each resource r used by A_i where the cost of the resource r depends on the congestion $f_r(n_r)$ defined by the number of agents using the same resource r . Thus, the goal of each agent is to minimize his penalty function. The best way to minimize his penalty is, in first place, to find a winning strategy and, second, to use the resources with minimum congestion.

Searching for an optimal interactive strategy is a hard problem because its effectiveness depends mostly on the strategies of the other agents involved. The agents are autonomous, however. Hence, their strategies are private [2].

3 An example for selecting a candidate in at University

For example, consider the problem of competing for selecting the best candidate to occupy an academic position at University. Each agent proposes a set of strategies that according to his beliefs are adequate to win the position. Let us assume that the group of candidates is formed by: $A = \{A(\text{Adrian}), J(\text{Juan}), M(\text{Mariana}), O(\text{Oscar})\}$. The agents compete for the position using a set of resources-influences which are appropriate for this responsibility. The set of resources each agent will use to build his own set of strategies, are shown in table 1.

In this case we have: $R = \{r_1, r_2, r_3, r_4\}$ with weights $W' = \{50, 20, 30, 20\}$ and $W_k = \sum_{i=1}^m W'_i = 50 + 20 + 30 + 20 = 130$.

Table 1. Resources and weights available for the Agents.

Resource r_i	Description r_i	W_i
r_1 :	To fulfil the requirements of the convocation	50
r_2 :	Experience in the area	20
r_3 :	Support of a chairman	30
r_4 :	Cover the background for the position	20

The conditions for the game, are:

- Each agent can only define 3 strategies as maximum.
- Each strategy can only consider 3 resources as maximum.
- Each agent can only define 4 constraints (2 oppositions and 2 supports beliefs) as maximum and the sum of the weights should be smaller than 100.

In Table 2 we show the resources that use each agent.

Table 2. Resources that use each agent.

	r_1	r_2	r_3	r_4
A	x	x	x	x
J		x	x	x
M	x	x	x	
O	x	x	x	x

Let us assume that each agent A_i $i=1, \dots, 4$ determines his set of strategies s_i as:

A: $s_1 = \{s_{11}, s_{12}, s_{13}\} = \{\{r_1, r_2, r_3\}, \{r_1, r_4\}, \{r_3, r_4\}\}$

J: $s_2 = \{s_{21}, s_{22}, s_{23}\} = \{\{r_2, r_3\}, \{r_2\}, \{r_4\}\}$

M: $s_3 = \{s_{31}, s_{32}, s_{33}\} = \{\{r_1, r_3\}, \{r_2, r_3\}, \{r_3\}\}$

O: $s_4 = \{s_{41}, s_{42}, s_{43}\} = \{\{r_2, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_4\}\}$

The principal of the university determines some characteristics that a candidate has to choose in order to cover the competing position. Such set of characteristics (resources), are: $C_k = \{\{r_2, r_4\}, \{r_1, r_3\}, \{r_1, r_2, r_3\}\}$. in Figure 1 we show the congestion network for this example.

Each agent selects one of his strategies forming the initial state: $e_1 = (s_{11}, s_{21}, s_{32}, s_{43})$; A_1 selects his strategy 1, A_2 selects his strategy 1, and so on.

Each one of them establishes a set of supports and oppositions for the others. The constraints and their associated weights, given by them, are:

Adrian \rightarrow \neg Mariana (30), Adrian \rightarrow \neg Oscar (33), \neg Adrian \rightarrow Mariana (20)

Juan \rightarrow \neg Adrian (40), \neg Juan \rightarrow Oscar (20).

Mariana \rightarrow \neg Oscar (35), Mariana \rightarrow \neg Adrian (35), \neg Mariana \rightarrow Adrian (30),

Oscar \rightarrow \neg Adrian (45), \neg Oscar \rightarrow Juan (50)

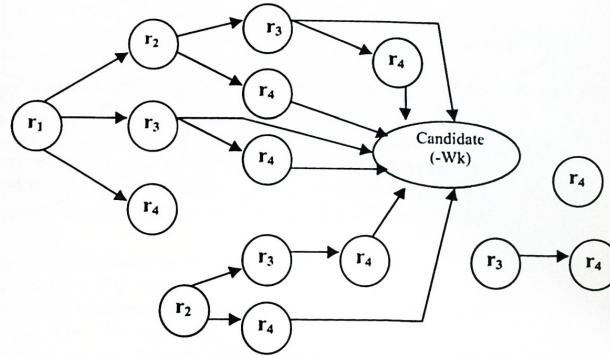


Fig. 2. A congestion network.

This show in the Table 3.

Table 3. The set of support and opposition for each agent.

Agent	Support				Opposition			
	A	J	M	O	$\neg A$	$\neg J$	$\neg M$	$\neg O$
A							30	33
J					40			
M					35			35
O					45			
$\neg A$			20					
$\neg J$				20				
$\neg M$	30							
$\neg O$		50						
	30	50	20	20	120	0	30	68

Now we compute the "Popularity" for each agent $A_i \in A$, according to our example, we have that:

$\text{Support}(A) = \sum_{e \in \text{in_edges}(A)} W(e) = 30$,
 $\text{Opposition}(A) = \sum_{e \in \text{in_edges}(\neg A)} W(e) = 40 + 35 + 45 = 120$
 $\text{Suit}(A) = \text{Support}(A) - \text{Opposition}(A) = -90$.
 $\text{Support}(J) = 50$, $\text{Opposition}(J) = 0$ and $\text{Suit}(J) = 50$.
 $\text{Support}(M) = 20$, $\text{Opposition}(M) = 30$ and $\text{Suit}(M) = -10$.
 $\text{Support}(O) = 20$, $\text{Opposition}(O) = 33+35=68$ and $\text{Suit}(O) = -48$.

We illustrate the dependence graph of those constraints in Figure 2.

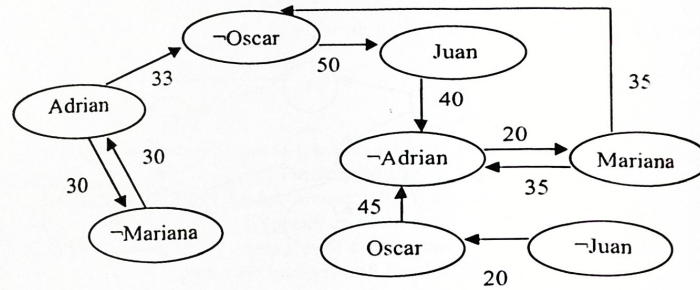


Fig. 2. The belief's graph of the multi-agent system.

The congestion function f_r for each resource, is determined as:

$$\begin{aligned}
 f_1(n_1) &= n_1 * w_1 = 1 * 50 = 50 \\
 f_2(n_2) &= n_2 * w_2 = 4 * 20 = 80 \\
 f_3(n_3) &= n_3 * w_3 = 3 * 30 = 90 \\
 f_4(n_4) &= n_4 * w_4 = 1 * 20 = 20
 \end{aligned}$$

And the *Opposition* for each agent, is:

$$\text{Opposition}(A_i) = \sum_{r \in S_i} f_r(n_r) + \text{reach}(s_i) - \text{suit}(A_i), \text{ then:}$$

$$\begin{aligned}
 \text{Opposition}(A_1) &= 50 + 80 + 90 - 130 - (-90) = 90 + 90 = 180 \\
 \text{Opposition}(A_2) &= 80 + 90 + 0 - 50 = 170 - 50 = 120 \\
 \text{Opposition}(A_3) &= 80 + 90 + 0 - (-10) = 170 + 10 = 180 \\
 \text{Opposition}(A_4) &= 80 + 20 + 0 - (-48) = 100 + 48 = 148
 \end{aligned}$$

The sum of results of $\text{Opposition}(A_i)$ for the action one, is: $\text{Sum} = 180 + 120 + 180 + 148 = 628$. For this state, we can see that the agent A_2 obtains the smallest opposition.

After some iterations over the space of states, where the agents change their strategies in order to take advantages and reduce their opposition function, we could

achieve the following state: $e_2 = (s_{12}, s_{23}, s_{31}, s_{42})$, where the following values for the congestion functions are:

$$f_1(n_1) = n_1 * w_1 = 3 * 50 = 150$$

$$f_2(n_2) = n_2 * w_2 = 0 * 20 = 0$$

$$f_3(n_3) = n_3 * w_3 = 2 * 30 = 60$$

$$f_4(n_4) = n_4 * w_4 = 3 * 20 = 60$$

Each agent establishes newly a set of supports and oppositions for the other ones, updating the table 3. The constraints and their associated weights, given by them, are:

Adrian \rightarrow \neg Oscar (25), Adrian \rightarrow Juan (40), \neg Adrian \rightarrow Mariana (20)

Juan \rightarrow \neg Adrian (60), \neg Juan \rightarrow Oscar (30), \neg Juan \rightarrow Mariana (10),

Mariana \rightarrow \neg Adrian (30), Mariana \rightarrow \neg Oscar (35), \neg Mariana \rightarrow Juan (30),

Oscar \rightarrow \neg Adrian (45), \neg Oscar \rightarrow Juan (50)

Now we compute the "Popularity" for each agent $A_i \in A$, according to our example, we have:

$$Support(A) = 0, Opposition(A) = 60 + 30 + 45 = 135 \text{ and } Suit(A) = -135.$$

$$Support(J) = 50 + 30 = 80, Opposition(J) = 40 \text{ and } Suit(J) = 40.$$

$$Support(M) = 20 + 10 = 30, Opposition(M) = 0 \text{ and } Suit(M) = 30.$$

$$Support(O) = 30, Opposition(O) = 25 + 35 = 60 \text{ and } Suit(O) = -30.$$

The opposition function for each agent, is:

$$Opposition(A_1) = 150 + 60 + 0 - 135 = 75$$

$$Opposition(A_2) = 60 + 0 - 40 = 20$$

$$Opposition(A_3) = 150 + 60 - 130 - 30 = 50$$

$$Opposition(A_4) = 150 + 60 + 60 - 130 - (-30) = 170$$

And the sum of results for $Opposition(A_i)$ for this state, is: $Sum = 75 + 20 + 50 + 170 = 315$. Notice that the agent A_2 when changing their strategy obtains the smallest value. Furthermore, A_2 has reached the candidacy of the group.

4 Conclusion

We have designed a logical model for attacking the problem of selecting a representative of a multi-agent system.

In our proposal, we assume that the candidacy of an agent depends on a set of 'influences-resources' in such a way that the candidacy will be for the agent which gets a minimum penalty for the strategy that he uses in order to obtain the candidacy and besides, the best candidate agent is the one whose candidacy does not generate a great opposition with the beliefs of the other agents. So, we prefer consistency in the candidacy of the agent more than popularity. This case is an example of a congestion game so, we can obtain a pure Nash equilibrium and then determine the agent with the minimum penalty who will be the best candidate of the group of agents.

The problem of selecting candidates is a relevant practical problem, for example, choosing candidates from political parties, selecting a department's boss, selecting a

supervisor, etc. In general, whenever it is necessary to select the best candidate (or the best candidates) from a group of people.

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